Chapter 16
Summary of Vector Calculus Concepts

Parameterizing Curves

		$\Re^3$	$\Re^2$
Parameterization of Curve *C*	$\vec{r}(t)$	$\langle f(t), g(t), h(t) \rangle$	$\langle f(t), g(t) \rangle$
Derivative of *C*	$\vec{r}\,'(t)$	$\langle f\,'(t), g\,'(t), h\,'(t) \rangle$	$\langle f\,'(t), g\,'(t) \rangle$
	$\lvert \vec{r}\,'(t) \rvert$	$\sqrt{[f\,'(t)]^2 + [g\,'(t)]^2 + [h\,'(t)]^2}$	$\sqrt{[f\,'(t)]^2 + [g\,'(t)]^2}$
Unit Tangent of *C*	$\vec{T}(t)$	$\dfrac{\vec{r}\,'(t)}{\lvert \vec{r}\,'(t) \rvert}$	
Unit Normal of *C*	$\vec{n}(t)$	**Not Applicable. i.e. no unique unit normal for Curve *C* in 3-Space**	$\dfrac{\langle g\,'(t), -f\,'(t) \rangle}{\lvert \vec{r}\,'(t) \rvert}$

Operations on Scalar Fields f & Vector Fields $\vec{F}$

		$\Re^3$	$\Re^2$
$\vec{F}$		$\langle P(x,y,z),\ Q(x,y,z),\ R(x,y,z) \rangle$	$\langle P(x,y),\ Q(x,y) \rangle$
Del operator or $\vec{\nabla}$		$\left\langle \dfrac{\partial}{dx}, \dfrac{\partial}{dy}, \dfrac{\partial}{dz} \right\rangle$	$\left\langle \dfrac{\partial}{dx}, \dfrac{\partial}{dy} \right\rangle$
Gradient $f(x,y)$ $grad\ f$ **or** $\vec{\nabla} f$		$\left\langle \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{dz} \right\rangle$	$\left\langle \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y} \right\rangle$
Divergence $\vec{F}(x,y)$ $div\ \vec{F}$ **or** $\vec{\nabla} \cdot \vec{F}$		$\dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z}$	$\dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y}$
If $\vec{F} = \vec{\nabla} f$ **(i.e** $\vec{F}$ **is *conservative*), then**		$\vec{\nabla} \times \vec{F} = 0$	$\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 0$
$curl\ \vec{F} = \vec{\nabla} \times \vec{F}$	$\Re^3$	$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left\langle \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z} \right), \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x} \right), \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \right) \right\rangle$	
	$\Re^2$	$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix} = \left\langle 0,\ 0,\ \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \right) \right\rangle$	

Line Integrals on the Parameterized Curve C ($C:\ \vec{r}(t)$ where $a \le t \le b$)		
Scalar Field $f(x,y,z)$	**Standard Form**	$\int_C f(x,y)ds = \int_a^b f(\vec{r}(t))\left\vert \vec{r}\,'(t)\right\vert dt$ **Note:** $ds = \left\vert \vec{r}\,'(t)\right\vert dt$
Vector Field $\vec{F}(x,y,z)$	**Standard Form**	$\int_C \vec{F}(x,y)\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$ **Note:** $d\vec{r} = \vec{r}\,'(t)\,dt$
	If Vector Field is of Form $\vec{F}(x,y) = \langle P(x,y),\ Q(x,y)\rangle$	$\int_C \vec{F}(x,y)\cdot d\vec{r} = \int_C Pdx + Qdy$
	Fundamental Theorem of Line Integrals **(Applies if** $\vec{F}$ **is conservative, i.e.** $\vec{F} = \nabla f$ **)**	$\int_C \vec{F}(x,y)\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$
		If C is a simple closed curve, $\int_C \vec{F}\cdot d\vec{r}$ **=0**
	Green's Theorem **If *C* is a simple closed curve enclosing domain *D* in a positive (counter-clockwise) direction and** $\vec{F}(x,y) = \langle P(x,y),\ Q(x,y)\rangle$	$\int_C \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$
	Vector Form of Green's Theorem	$\int_C \vec{F}\cdot d\vec{r} = \iint_D (curl\,\vec{F})\cdot \vec{k}\,dA$
	Stokes' Theorem (Greens Theorem in 3D) **If C is a simple closed curve with positive orientation that bounds a smooth surface and** $\vec{F}(x,y,z) = \langle P(x,y,z),\ Q(x,y,z),\ R(x,y,z)\rangle$	$\int_C \vec{F}\cdot d\vec{r} = \iint_S (curl\,\vec{F})\cdot d\vec{S}$ **Note:** $d\vec{S} = \vec{n}\,dA$ **where** $\vec{n}$ **is the normal to the surface D is the domain resulting from the projection of *C* onto the *xy*-plane**
	Line Integral of the normal component ($\vec{n}$ **) of** $\vec{F}$ **along *C*.**	$\int_C \vec{F}(x,y)\cdot \vec{n}\,ds = \iint_D (div\,\vec{F})\,dA$

Surface Integrals on the Parameterized Surface S ($S:\ \vec{r}(u,v)$ where $a \le u \le b$, and $c \le v \le d$)		
Scalar Field $f(x,y,z)$	**Standard Form**	$\iint_S f\,dS = \iint_D f(\vec{r}(u,v))\left\vert \vec{r}_u \times \vec{r}_v\right\vert dA$
Vector Field $\vec{F}(x,y,z)$	**Standard Form**	$\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r}_v)\,dA$
	Divergence Theorem **If the surface *S* is the boundary for a simple solid region *E* (with positive outward orientation).**	$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E div\ \vec{F}\,dV$

Basic Steps to Solve Line/Surface Integral					
		Line Integrals		**Surface Integrals**	
1. Parameterize		$C = \vec{r}(t)\ a \le t \le b$		$S = \vec{r}(u,v)\ a \le u \le b,\ c \le v \le d$	
2. Express Field in Terms of Parameterization		**Scalar Field**	**Vector Field**	**Scalar Field**	**Vector Field**
		$f(\vec{r}(t))$	$\vec{F}(\vec{r}(t))$	$f(\vec{r}(u,v))$	$\vec{F}(\vec{r}(u,v))$
3. Evaluate 2nd Part of Integrand		$ds = \left\vert \vec{r}\,'(t)\right\vert dt$	$d\vec{r} = \vec{r}\,'(t)dt$	$dS = \left\vert \vec{r}_u \times \vec{r}_v\right\vert dA$	$d\vec{S} = (\vec{r}_u \times \vec{r}_v)dA$
4. Put the integral together	**Scalar Field**	$\int_a^b f(\vec{r}(t))\left\vert \vec{r}\,'(t)\right\vert dt$		$\int_c^d \int_a^b f(\vec{r}(u,v))\left\vert \vec{r}_u \times \vec{r}_v\right\vert dudv$	
	Vector Field	$\int_a^b \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$		$\int_c^d \int_a^b \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r})\,dudv$	